

# Lesson 26 - Functions of Several Variables

- Evaluating*
- I. Functions of Several Variables
  - II. Domain and Range
  - III. Level Curves
- 

Exam 3  
See  
memo  
in  
Brightspace

## Exam 3 ▼

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Posted Oct 30, 2023 10:38 AM

Exam 3 is Wednesday, November 8 from 6:30pm-7:30pm. The location of the exam will depend on your instructor (and please note that these locations are different than the first 2 exams):

Ben Doyle: LILY 1105

Victor Hughes: CL50 224

Jakayla Robbins: LILY 1105

New room!

Alexandra Cuadra: CL50 224

Dave Norris: CL50 224

There is an exam memo in Brightspace under Contents->Exam Information with more detailed information about the exam. You will be emailed a seating assignment closer to the exam date.

Attachment(s):

 Exam3Memo.pdf

(41.65 KB)

## I. Evaluating $f(x, y)$

[Ex] Compute the function value

$$a) f(x, y) = \ln(xy)$$

$$f(e^2, e^3) = \ln(e^2 \cdot e^3) = \ln(e^5) = 5$$

$$b) f(x, y) = \sin(x+y)$$

$$f\left(\frac{\pi}{3}, \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = \textcircled{1}$$
$$\frac{\frac{2\pi}{6} + \frac{\pi}{6}}{\frac{3\pi}{6}} = \frac{\pi}{2}$$

## II. Domain and Range

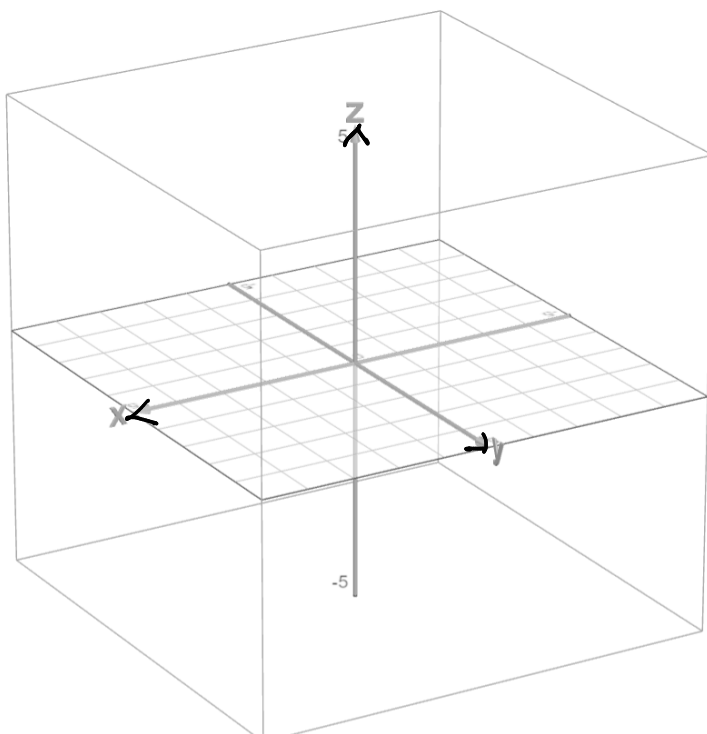
↑  
set of  
allowable  
inputs

↑  
set of  
possible  
outputs

Axis system for fctns of 2 variables  
If you want to graph  $f(x, y)$

$$z = f(x, y)$$

xyz-space



To graph  $f(x, y)$ , we graph  $z = f(x, y)$

So if  $f(x, y) = x^2 - y$ . Then some points

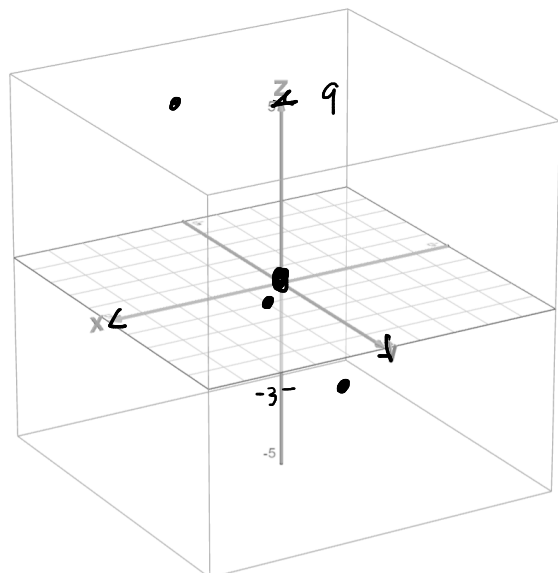
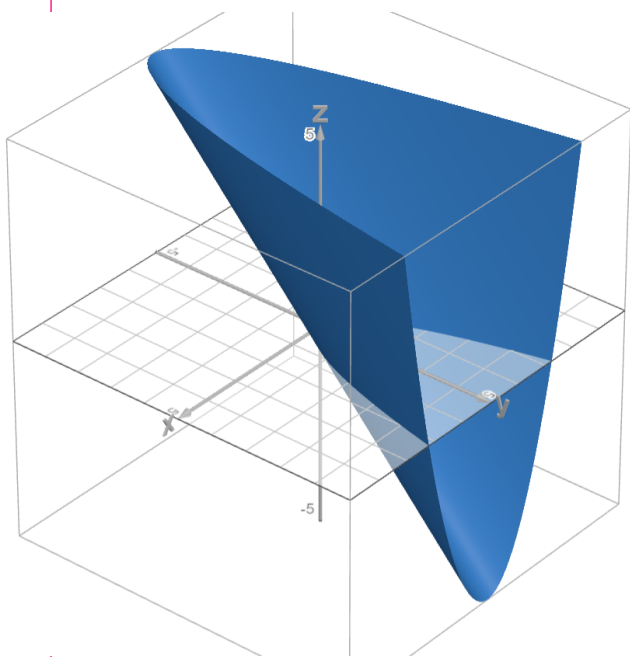
on the graph of this function would be

$$(0, 0, 0)$$

$$(1, 1, 0)$$

$$(0, 3, -3)$$

$$(3, 0, 9)$$



← graph of  $z = f(x, y)$   
 $= x^2 - y$

Domain For  $f(x, y)$

Domain of  $f$ :  $D_f = \{(x, y) \mid f(x, y) \text{ is a real \#}\}$

$$\boxed{\text{Ex}} \quad h(x, y) = \sqrt{\underbrace{x^2 + y^2 - 4}_{\geq 0}}$$

$$D_h = \{(x, y) \mid x^2 + y^2 - 4 \geq 0\}$$
$$= \{(x, y) \mid x^2 + y^2 \geq 4\}$$

Things to watch for with domain

- (1) Don't divide by 0
- (2) Don't put negatives under even roots  $\sqrt{\quad}$ ,  $\sqrt[4]{\quad}$ ,  $\sqrt[6]{\quad}$ , etc  
input  $\geq 0$
- (3) Don't put 0 or negatives in logs  
input  $> 0$

Range: The set of all possible  $z$  values

$$h(x,y) = \sqrt{x^2 + y^2 - 4}$$

$$R_h: [0, \infty)$$

Things to consider about range:

$$(1) R_{e^x} : (0, \infty)$$

$$(2) R_{\ln(x)} : (-\infty, \infty)$$

$$(3) R_{\substack{\sin(x) \\ \text{or} \\ \cos(x)}} : [-1, 1]$$

$$(4) R_{\sqrt{x}} : [0, \infty)$$

Ex Find the domain of  $f_2(x,y) = \frac{\ln(x-2)}{\sqrt{y-1}}$

$$\begin{array}{l} \ln \\ x-2 > 0 \\ x > 2 \end{array}$$

$$\begin{array}{l} \sqrt{\quad} \\ y-1 \geq 0 \\ y \geq 1 \end{array}$$

$$\begin{array}{l} \text{Denom.} \\ \sqrt{y-1} \neq 0 \\ y-1 \neq 0 \\ y \neq 1 \end{array}$$

$$y > 1$$

$$D_{f_2}: \{(x, y) \mid x > 2 \text{ and } y > 1\}$$

## II. Level Curves

Like contour maps.

① A **level curve** of  $f(x, y)$  is the graph of

$$f(x, y) = C$$

for some constant  $C$

Notice: No  $z$ .  
This graph is in  $xy$ -plane

**Ex** Describe the level curves

a)  $f(x, y) = 2\sqrt{x-5y^2}$

$$2\sqrt{x-5y^2} = C$$
$$\sqrt{x-5y^2} = C/2$$

$$x-5y^2 = (C/2)^2$$

$$x-5y^2 = k$$

sideways parabola. Only  $y^2$

b)  $f(x, y) = 2(x-4)^2 + 2(y+3)^2$

$$C = 2(x-4)^2 + 2(y+3)^2$$

$$(x-4)^2 + (y+3)^2 = \frac{C}{2}$$

circle, radius  $\sqrt{\frac{C}{2}}$  center  $(4, -3)$

$x^2, y^2$   
same  $^2$   
for both